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JEE (Main)

PAPER-1 (B.E./B. TECH.)

2021

COMPUTER BASED TEST (CBT)
Memory Based Questions & Solutions

Date: 25 July, 2021 (SHIFT-1) | TIME : (9.00 a.m. to 12.00 p.m)

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SUBJECT: MATHEMATICS

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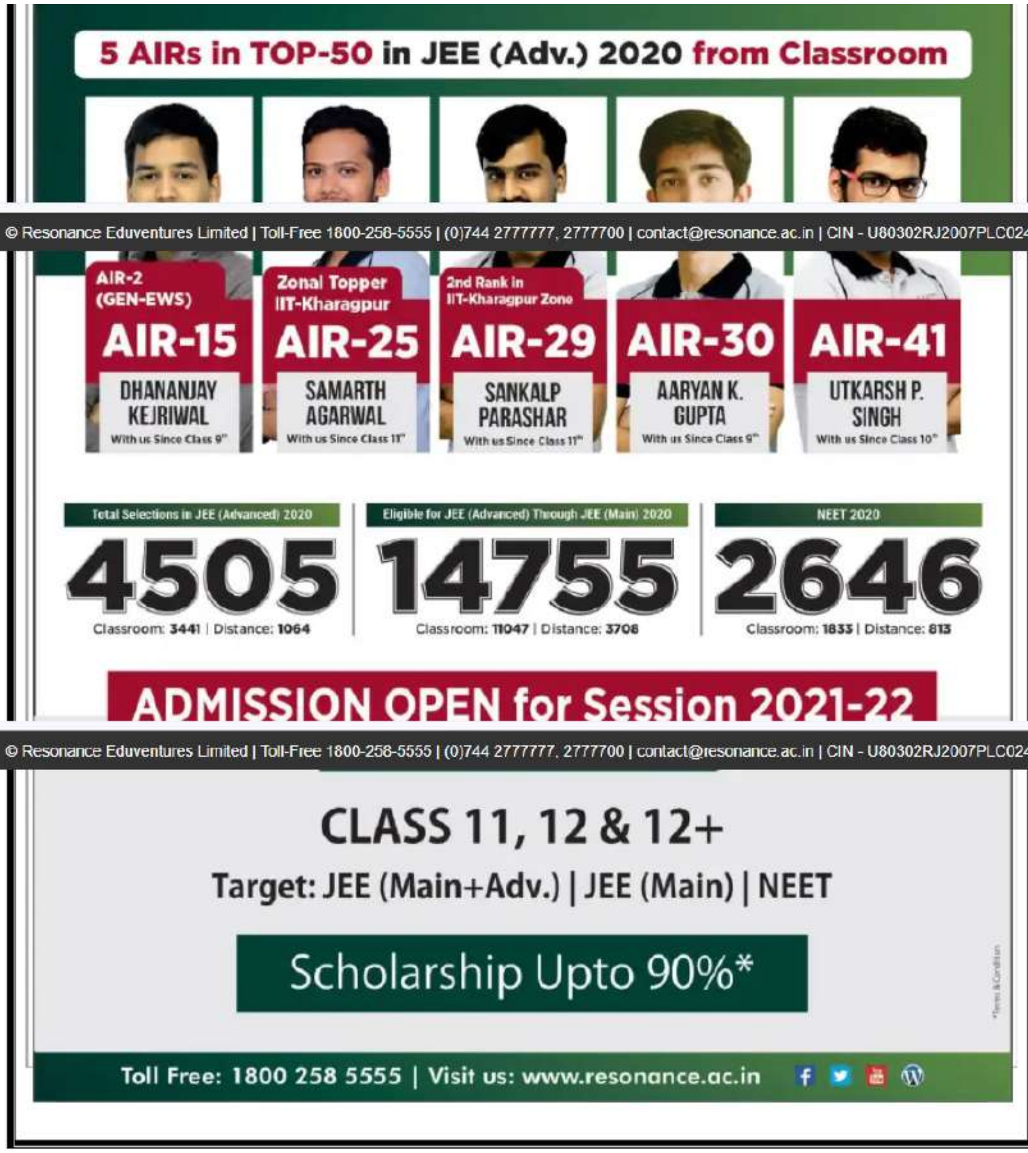

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PART : MATHEMATICS

1 If the ratio of coefficient of middle term in the expansion of $(1 + x)^{20}$ and sum of coefficients of middle terms in the expansion of $(1 + x)^{19}$ is λ, then λ is :

(1) 2 (2) 1 (3) 3 (4) 4

Ans. (2)

Sol. $\dfrac{{}^{20}C_{10}}{{}^{19}C_9 + {}^{19}C_{10}} = \dfrac{{}^{20}C_{10}}{{}^{20}C_{10}} = 1$

2. If $f(x) = \begin{cases} \dfrac{\lambda}{\mu}\dfrac{|x^2 - 5x + 6|}{5x - 6 - x^2} & ; x < 2 \\ \mu & ; x = 2 \\ e^{\frac{\tan(x-2)}{x-[x]}} & ; x > 2 \end{cases}$, is continuous at $x = 2$. Then the sum of λ and μ is:

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Sol. $\text{RHL} = \lim\limits_{x \to 2^+} e^{\frac{\tan(x-2)}{x-[x]}} = \lim\limits_{x \to 2^+} e^{\frac{\tan(x-2)}{(x-2)}} = e$

$\text{LHL} = \lim\limits_{x \to 2^-} \dfrac{\lambda}{\mu}\dfrac{|x^2 - 5x + 6|}{5x - 6 - x^2}$

For $x < 2$, $|x^2 - 5x + 6| = x^2 - 5x + 6$

$\therefore \text{LHL} = \lim \dfrac{\lambda}{\mu}\dfrac{|x^2 - 5x + 6|}{\ldots} = \dfrac{-\lambda}{\ldots}$

$\therefore LHL = \lim_{x \to 2^-} \mu(5x - 6 - x^2) = \mu$

Also, $f(2) = \mu$

For f(x) to be continuous at x = 2,

RHL = LHL = f(2)

$\therefore e = \frac{-\lambda}{\mu} = \mu$

$\Rightarrow \mu = e$ and $\lambda = -e^2$

$\therefore \lambda + \mu = e - e^2$

3. If $\sin x + \sin 2x + \sin 3x + \sin 4x = 0$, $x \in [0, 2\pi]$ then sum of values of x is

(1) 7π (2) 9π (3) 11π (4) 12π

Ans. (2)

Sol. $(\sin x + \sin 4x) + (\sin 2x + \sin 3x) = 0$

$\Rightarrow 2\sin\frac{5x}{2}\cos\frac{3x}{2} + 2\sin\frac{5x}{2}\cos\frac{x}{2} = 0$

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$\Rightarrow 2\sin\frac{5x}{2}\left(\cos\frac{3x}{2} + \cos\frac{x}{2}\right) = 0$

$\Rightarrow 4\sin\frac{5x}{2}\cos x\cos\frac{x}{2} = 0$

$\Rightarrow \sin\frac{5x}{2} = 0$ or $\cos x = 0$ or $\cos\frac{x}{2} = 0$

$\Rightarrow \frac{5x}{2} = 0, \pi, 2\pi, 3\pi, 4\pi, 5\pi$ or $x = \frac{\pi}{2}, \frac{3\pi}{2}$ or $\frac{x}{2} = \frac{\pi}{2}$

$\Rightarrow x = 0, \frac{2\pi}{5}, \frac{4\pi}{5}, \frac{6\pi}{5}, \frac{8\pi}{5}, \frac{10\pi}{5}$ or $x = \frac{\pi}{2}, \frac{3\pi}{2}$ or $x = \pi$

$\Rightarrow x = 0, \frac{2\pi}{5}, \frac{4\pi}{5}, \frac{6\pi}{5}, \frac{8\pi}{5}, 2\pi, \frac{\pi}{2}, \frac{3\pi}{2}, \pi$

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4. The number of real solutions of the equation $e^{6x} + e^{4x} + 2e^{3x} + 12e^{2x} + e^x - 1 = 0$

(1) 0 (2) 1 (3) 6 (4) 8

Ans. (2)

Sol. Since $f(x) = e^{6x} + e^{4x} + 2e^{3x} + 12e^{2x} + e^x - 1$

$\Rightarrow f'(x) = 6e^{6x} + 4e^{4x} + 6e^{3x} + 24e^{2x} + e^x > 0, \ \forall \ x \in R$

Hence f(x) is an increasing function

Now $\lim_{x \to -\infty} f(x) = -1$ and $f(0) = 1 + 1 + 2 + 12 + 1 - 1$

$\Rightarrow f(0) > 0$

Hence $f(x) = 0$ has a root in $(-\infty, 0)$

5. If $\frac{1}{a-b} + \frac{1}{a-2b} + \ldots\ldots\frac{1}{a-nb} = \alpha n + \beta n^2 + \gamma n^3$ where a is so large than b such that cube and higher powers of $\frac{b}{a}$ may be neglected then value of γ is :

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(1) $\frac{b^2}{3a^3}$ (2) $\frac{a-b}{3a^3}$ (3) $\frac{a-b}{3a^2}$ (4) $\frac{b}{2a^2}$

Ans. (1)

Sol. $\frac{1}{a-b} + \frac{1}{a-2b} + \frac{1}{a-3b} + \ldots\ldots + \frac{1}{a-nb}$

$$=\frac{1}{a}\left[\left(1-\frac{b}{a}\right)^{-1}+\left(1-\frac{2b}{a}\right)^{-1}+\left(1-\frac{3b}{a}\right)^{-1}+\ldots\ldots+\left(1-\frac{nb}{a}\right)^{-1}\right]$$

$$=\frac{1}{a}\left[\left\{1+\left(\frac{b}{a}\right)+\left(\frac{b}{a}\right)^2+\ldots\ldots\right\}+\left\{1+\left(\frac{2b}{a}\right)+\left(\frac{2b}{a}\right)^2+\ldots\ldots\right\}+\left\{1+\left(\frac{nb}{a}\right)+\left(\frac{nb}{a}\right)^2+\ldots\ldots\right\}\right]$$

$$=\frac{1}{a}\left[n+\frac{b}{a}(1+2+\ldots\ldots+n)+\frac{b^2}{a^2}(1^2+2^2+\ldots\ldots n^2)\right]$$

$$=\frac{1}{a}\left[n+\frac{n(n+1)}{2}\frac{b}{a}+\frac{n(n+1)(2n+1)}{6}\frac{b^2}{a^2}\right]$$

$$=\frac{1}{a}\left[n+\frac{n^2}{2}\frac{b}{a}+\frac{n}{2}\frac{b}{a}+\frac{2n^3+3n^2+n}{6}\left(\frac{b^2}{a^2}\right)\right]$$

$$=n\left(\frac{1}{a}+\frac{b}{2a^2}+\frac{b^2}{6a^3}\right)+\left(\frac{b}{2a^2}+\frac{b^2}{2a^3}\right)n^2+\frac{b^2}{3a^3}n^3$$

we get $\gamma=\frac{b^2}{3a^3}$

6 In an A.P, if $S_{3n} = 3S_{2n}$, then ratio $\frac{S_{4n}}{S_{2n}}$ equals:

(1) 8 (2) 6 (3) 4 (4) 2

Ans. (2)

Sol. $\frac{S_{3n}}{S_{2n}}=\frac{\frac{3n}{2}[2a+(3n-1)d]}{\frac{2n}{2}[2a+(2n-1)d]}=3$

$\Rightarrow 2a+(3n-1)d = 2[2a + (2n-1)d]$

$\Rightarrow 2a+(n-1)d = 0$ ......(1)

Now $\frac{S_{4n}}{S_{2n}}=\frac{\frac{4n}{2}[2a+(4n-1)d]}{\frac{2n}{2}[2a+(2n-1)d]}$

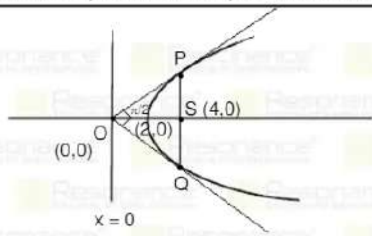
$[2a+(2n-1)d]$

Put, $2a = -(n-1)d$, we have, $\frac{S_{4n}}{S_{2n}}=\frac{2[3nd]}{nd}=6$

7. A parabola whose vertex is at 2 unit distance from origin on positive x-axis and distance between focus and origin is 4 unit. The tangent drawn from the origin to the parabola meet the parabola at P and Q, then the area of ΔOPQ is

Ans. **16 square unit**

Sol.



Equation of parabola

$(y-0)^2 = 4(2)(x-2)$

origin lie on directrix x = 0

$T = 0$

$x = 4$

It is latus rectum of parabola

so area of OPQ = $\frac{1}{2} \times 4 \times 8 = 16$ square unit

8. The term independent of x in expansion of $\left(\frac{x+1}{x^{2/3}-x^{1/3}+1} - \frac{x-1}{x-x^{1/2}}\right)^{10}$ is :

(1) 4 (2) 120 (3) 210 (4) 310

Sol. (3)

$\left((x^{1/3}+1) - \left(\frac{\sqrt{x}+1}{\sqrt{x}}\right)\right)^{10}$

$(x^{1/3} - x^{-1/2})^{10}$

$T_{r+1} = {}^{10}C_r\,(x^{1/3})^{10-r}\,(-x^{-1/2})^r$

$\frac{10-r}{3} - \frac{r}{2} = 0 \Rightarrow \quad 20 - 2r - 3r = 0$

$\Rightarrow \quad r = 4$

$T_5 = {}^{10}C_4 = \frac{10\times 9\times 8\times 7}{4\times 3\times 2\times 1} = 210$

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(1) $\frac{11}{12}$ (2) $\frac{7}{3}$ (3) $\frac{2}{3}$ (4) $\frac{5}{2}$

Ans. (2)

Sol.

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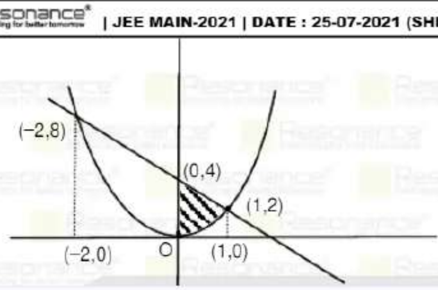

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$2x^2 = 4 - 2x$
$x^2 + x - 2 = 0$
$(x + 2)(x - 1) = 0$
$x = -2, x = 1$

Required area = $\frac{1}{2}(2+4)\times 1 - \int_0^1 2x^2 dx = 3 - \frac{2}{3} = \frac{7}{3}$ square units

10. An ellipse with eccentricity $\frac{1}{\sqrt{3}}$ passes through the point $\left(\sqrt{\frac{3}{2}}, 1\right)$. A circle is drawn whose centre is the focus of ellipse and its radius is $\frac{2}{\sqrt{3}}$. If circle cuts the ellipse at two different points P and Q then the value of PQ^2 is

(1) $\frac{8}{3}$ (2) $\frac{4}{3}$ (3) $\frac{16}{3}$ (4) $\frac{5}{3}$

Sol. Let equation of ellipse is $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ $(a > b)$

it passes through $\left(\sqrt{\frac{3}{2}}, 1\right) \Rightarrow \frac{3}{2a^2} + \frac{1}{b^2} = 1$ ....(1)

Given $e = \frac{1}{\sqrt{3}} \Rightarrow b^2 = a^2(1 - e^2) = \frac{2}{3}a^2$ ....(2)

Solve (1) & (2) we get $a^2 = 3, b^2 = 2$

$\therefore$ Ellipse is $\frac{x^2}{3} + \frac{y^2}{2} = 1$ .... (3)

Focus $(\pm ae, 0) = \left(\pm\sqrt{3}.\frac{1}{\sqrt{3}}, 0\right) = (\pm 1, 0)$

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Hence circle is $(x - 1)^2 + y^2 = \left(\frac{2}{\sqrt{3}}\right)^2 = \frac{4}{3}$ ....(4)

Solve (3) & (4)

$2x^2 + 3\left(\frac{4}{3} - (x-1)^2\right) = 6$

$2x^2 + 4 - 3(x^2 + 1 - 2x) = 6$

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$x = 1, 5$

When $x = 1 \Rightarrow \frac{1}{3} + \frac{y^2}{2} = 1 \Rightarrow y^2 = \frac{4}{3} \Rightarrow y = \pm\frac{2}{\sqrt{3}}$

Hence $P\left(1, \frac{2}{\sqrt{3}}\right), Q\left(1, -\frac{2}{\sqrt{3}}\right) \Rightarrow PQ^2 = \frac{16}{3}$

When $x = 5 \Rightarrow \frac{y^2}{2} = 1 - \frac{25}{3} = -\frac{22}{3} \Rightarrow$ not possible

$PQ^2 = \frac{16}{3}$

11. The statement $(p \to q) \wedge (p \to \sim q)$ is logically equivalent to:
(1) p (2) q (3) ~ p (4) ~ q

Ans. (3)

Sol. $= (p \to q) \wedge (p \to \sim q)$
$= (\sim p \vee q) \wedge (\sim p \vee \sim q)$

$= \sim p \vee (q \wedge \sim q)$

12. In class 10^{th}, 11^{th} and 12^{th} of a school there are 5, 6 and 8 students respectively. The number of ways of selection of 10 students such that at least two students are selected from each of the classes and at most 5 students together can be selected from class 10^{th} & 11^{th} are $k \times 100$ then the value of k is.

Ans. 238

Sol.

Total student	(5)	(6)	(8)		
Class	10^{th}	11^{th}	12^{th}		
	2	2	6	$\rightarrow$	${}^5C_2 \times {}^6C_2 \times {}^8C_6$
	2	3	5	$\rightarrow$	${}^5C_2 \times {}^6C_3 \times {}^8C_5$
	3	2	5	$\rightarrow$	${}^5C_3 \times {}^6C_2 \times {}^8C_5$

Total number of ways $= {}^5C_2 \times {}^8C_3\left({}^6C_3 + {}^6C_2\right) + {}^5C_2 \times {}^6C_2 \times {}^8C_6$

$= 23800$

13. If $\left(1+\frac{2}{3}+\frac{6}{3^2}+\frac{10}{3^3}+......+\infty\right)^{\log_{0.25}\left(\frac{1}{3}+\frac{1}{3^2}+\frac{1}{3^3}+...\right)}$ is ℓ then ℓ^2 is

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Ans. (3)

Sol. $\left(1+\frac{2}{3}+\frac{6}{3^2}+\frac{10}{3^3}+......\right)^{\log_{0.25}\left(\frac{1}{3}\left(\frac{1}{\frac{2}{3}}\right)\right)} = \ell$

$\left(1+\frac{2}{3}+\frac{6}{3^2}+\frac{10}{...}\right)^{\log_{\frac{1}{4}}\frac{1}{2}} = \ell$

Let $1+\frac{2}{3}+\frac{6}{3^2}+\frac{10}{3^3}+..... = x$

$(x-1) = \frac{2}{3}+\frac{6}{3^2}+\frac{10}{3^3}+....$ .... (1)

$\frac{1}{3}(x-1) = \frac{2}{3^2}+\frac{6}{3^3}+.....$ .... (2)

From (1) – (2), we get

$\frac{2}{3}(x-1) = \frac{2}{3}+\frac{4}{3^2}+\frac{4}{3^3}+.....$

$\frac{2}{3}(x-1) = \frac{2}{3}+\frac{4}{3^2}\left(\frac{1}{1-\frac{1}{3}}\right)$

$\frac{2}{3}(x-1) = \frac{2}{3}+\frac{4}{3^2}\cdot\frac{3}{2}$

$\frac{2}{3}(x-1) = \frac{2}{3}+\frac{2}{3}$

$x - 1 = 2$ & $x = 3$

$3^{\frac{1}{2}} = \ell$

$\ell^2 = 3$

14. A hyperbola with equation $\frac{(x-1)^2}{16} - \frac{(y+2)^2}{9} = 1$ is given. A triangle is formed with two vertices as the focus of the hyperbola and third vertex lies on hyperbola. The locus of centroid of the triangle is:

(1) $16(x-1)^2 - 9(y+2)^2 = 16$
(2) $9(x-1)^2 - 16(y+2)^2 = 16$
(3) $9(x-1)^2 + 16(y+2)^2 = 16$
(4) $16(x-1)^2 + 9(y+2)^2 = 16$

Ans. (2)

Sol. Given $\frac{(x-1)^2}{16} - \frac{(y+2)^2}{9} = 1$

Let $x - 1 = X$

$y + 2 = Y$

$$\frac{X^2}{16}-\frac{Y^2}{9}=1$$

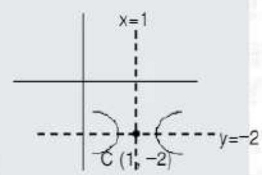
$b^2 = a^2(e^2-1) \Rightarrow \frac{9}{16} = e^2 - 1 \Rightarrow e = \frac{5}{4}$

Focus $(\pm ae, 0) \Rightarrow X = \pm ae,\ \ Y = 0$

Hence focus $S(-4, -2)$, $S'(6, -2)$

Let any point on hyperbola $x-1 = 4\sec\theta$, $y+2 = 3\tan\theta \Rightarrow P(1+4\sec\theta, -2+3\tan\theta)$

Hence centroid is $= \left(\frac{-4+6+1+4\sec\theta}{3}, \frac{-2-2-2+3\tan\theta}{3}\right)$



$h = \frac{3+4\sec\theta}{3} \Rightarrow \sec\theta = \frac{3h-3}{4}$

$k = \frac{-6+3\tan\theta}{3} \Rightarrow \tan\theta = k+2$

$\sec^2\theta - \tan^2\theta = 1$

$\left(\frac{3h-3}{4}\right)^2 - (k+2)^2 = 1$

Locus is $\frac{9(x-1)^2}{16} - \frac{(y+2)^2}{1} = 1$

$\Rightarrow 9(x-1)^2 - 16(y+2)^2 = 16$

15. Evaluate $\int_{\frac{\pi}{24}} \frac{dx}{1+\sqrt[3]{\tan 2x}}$:

(1) $\frac{\pi}{24}$ (2) $\frac{\pi}{4}$ (3) $\frac{\pi}{12}$ (4) $\frac{\pi}{6}$

Ans. (3)

Sol. $\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$

$$I = \int_{\frac{\pi}{24}}^{\frac{5\pi}{24}} \frac{dx}{1+\sqrt[3]{\tan 2x}} \ ......(1)$$

By property

$$I = \int_{\frac{\pi}{24}}^{\frac{5\pi}{24}} \frac{dx}{1+\sqrt[3]{\cot 2x}}$$

$I = \int^{\frac{5\pi}{24}} \frac{\sqrt[3]{\tan 2x}\ dx}{...}$ ......(2)

By adding (1) & (2)

$$2I = \int_{\frac{\pi}{24}}^{\frac{5\pi}{24}} \frac{(1+\sqrt[3]{\tan 2x})\ dx}{1+\sqrt[3]{\tan 2x}}$$

$$2I = \int_{\frac{\pi}{24}}^{\frac{5\pi}{24}} dx = \frac{\pi}{6}$$

$$\therefore I = \frac{\pi}{12}$$

16. If a set of matrix $M = \left\{\begin{pmatrix} a & b \\ c & d \end{pmatrix} : a,b,c,d \in \{\pm 1, \pm 2, \pm 3\}\right\}$ and $A \in M$ then the number of such matrices A whose determinant value is 15.

Ans. (16)

Case – I $ad = 9$ & $bc = -6$

$ad = (3,3)$ or $(-3,-3)$ $bc = (2,-3), (-2,3), (-3,2), (3,-2)$

Total $= 2 \times 4 = 8$ matrix

Case – II $ad = 6$ and $bc = -9$

Similarly, Total $= 4 \times 2 = 8$ matrix

Total such matrix $= 8 + 8 = 16$ matrix

17. If $\frac{dy}{dx} = 1 + xe^{y-x}$, $-\sqrt{2} < x < \sqrt{2}$ and $y(0) = 0$ then minimum value of y is

(1) $(1-\sqrt{3}) - \ell n(\sqrt{3}-1)$ (2) $(1+\sqrt{3}) - \ell n(\sqrt{3}-1)$

(3) $(1-\sqrt{3}) - \ell n(\sqrt{3}+1)$ (4) $(1+\sqrt{3}) - \ell n(\sqrt{3}+1)$

Ans. (1)

Sol. $\frac{dy}{dx} = 1 + xe^{y-x}$ ....(1)

$e^{-y}\frac{dy}{dx} = e^{-y} + xe^{-x}$

Put $e^{-y} = t \Rightarrow e^{-y}\frac{dy}{dx} = -\frac{dt}{...}$

dx

$I.F. = e^{\int 1.dx} = e^x$

Solution of equation (2) is

$te^x = \int (-xe^{-x}).e^x dx + c$

$te^x = -\frac{x^2}{2} + c$

$e^{x-y} = -\frac{x^2}{2} + c$ ....(3)

$\therefore y(0) = 0 \Rightarrow 1 = c \Rightarrow e^{x-y} = \left(\frac{2-x^2}{...}\right)$

$x - y = \ell n\left(\frac{2-x^2}{2}\right)$

$y = x - \ell n\left(\frac{2-x^2}{2}\right)$

Now, $\frac{dy}{dx} = 1 + x\left(\frac{2}{2-x^2}\right)$

$\Rightarrow -\left(\frac{x^2-2x-2}{2-x^2}\right) = 0$

$x = 1 \pm \sqrt{3}$

$-$ $\quad 1-\sqrt{3}$ $\quad +$ $\quad 1+\sqrt{3}$ $\quad -$

$\Rightarrow y_{min}$ at $x = 1 - \sqrt{3}$ $\quad \Rightarrow y_{min} = (1-\sqrt{3}) - \ell n(\sqrt{3}-1)$

18. A balloon with radius 16 cm is at a certain height above the ground such that the angle of elevation of its centre is 75° from a point on the ground. if the balloon subtends an angle of 60° at that point, then the height of its topmost point from the ground is:

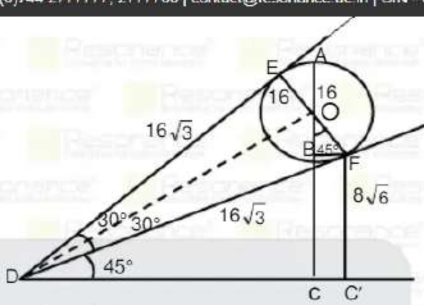
(1) $8\left(\sqrt{6}-\sqrt{2}+2\right)$ (2) $8\left(\sqrt{6}-2\sqrt{2}+2\right)$

(3) $8\left(\sqrt{6}+\sqrt{2}+2\right)$ (4) $8\left(\sqrt{6}-2\sqrt{2}+2\right)$

Ans. (3)

Sol. In triangle EOD

$ED = \frac{16}{\tan 30^\circ} = 16\sqrt{3}$



$DF = 16\sqrt{3}$

Now in Δ DFC′

$C'F = 16\sqrt{3}.\sin 45^\circ$

$= 16\sqrt{3}.\frac{1}{\sqrt{2}} = 8\sqrt{6}$

$CB = 8\sqrt{6}$

In Δ^{le} OBF

$OB = 16\sin 45^\circ$

$= \frac{16}{\sqrt{2}} = 8\sqrt{2}$

Height of top most point $= 8\sqrt{6} + 8\sqrt{2} + 16 = 8\left(\sqrt{6}+\sqrt{2}+2\right)$

19. If S = {n : n ∈ N and 1 ≤ n ≤ 100, $\begin{bmatrix} & \\ i & 0\end{bmatrix}\begin{bmatrix} & \\ c & d\end{bmatrix} = \begin{bmatrix} & \\ c & d\end{bmatrix}$ ∀ a,b,c,d ∈ R} then the number of 2 digit numbers in S is

(1) 25 (2) 22 (3) 24 (4) 20

Ans. (2)

Sol. Let A = $\begin{bmatrix}0 & i\end{bmatrix}^n$ and B = $\begin{bmatrix}a & b\end{bmatrix}$

Sol. Let A = $\begin{bmatrix} i & 0 \end{bmatrix}$ and B = $\begin{bmatrix} c & d \end{bmatrix}$

AB = IB

(A – I)B = 0

A = I

$\begin{bmatrix} 0 & i \\ i & 0 \end{bmatrix}^n = I$

$A^4 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

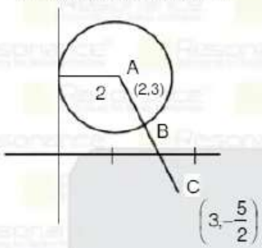
20. If $|z - (3i + 2)| < 2$ then the min value of $|2z - 6 + 5i|$ is

(1) $(5\sqrt{5} - 2)$ (2) $(5\sqrt{5} - 4)$ (3) $(5\sqrt{5} + 2)$ (4) $\left(\frac{5\sqrt{5}}{2} + 2\right)$

Ans. (2)

Sol. $|(x - 2) + (y - 3)i| < 2$

Point A (2,3) and radius = 2



$|(2x - 6) + (2y + 5)i|$

x = 3

5

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point C $\left(3, -\frac{5}{2}\right)$

$$AC = \sqrt{(3-2)^2 + \left(-\frac{5}{2} - 3\right)^2}$$

$$= \sqrt{1 + \frac{121}{4}}$$

$$= \sqrt{\frac{125}{4}}$$

Min distance $2BC = 2\left(\frac{5\sqrt{5}}{2} - 2\right) = (5\sqrt{5} - 4)$

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21. Let $\vec{p} = (3\hat{i} + 2\hat{j} + \hat{k})$, $\vec{q} = (2\hat{i} + \hat{j} + \hat{k})$ and $\vec{r}$ is perpendicular to both $\vec{p}+\vec{q}$ and $\vec{p}-\vec{q}$ such that $|\vec{r}| = \sqrt{3}$. If $\vec{r} = (a\hat{i} + b\hat{j} + c\hat{k})$, then the value of $|a| + |b| + |c|$ is :

(1) 0 (2) 1 (3) 3 (4) 6

Ans. (3)

Sol. $(\vec{p}+\vec{q}) \times (\vec{p}-\vec{q}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 5 & 3 & 2 \\ 1 & 1 & 0 \end{vmatrix} = -2\hat{i} + 2\hat{j} + 2\hat{k}$

$\vec{r} = \pm\sqrt{3}\,\dfrac{((\vec{p}+\vec{q}) \times (\vec{p}-\vec{q}))}{|(\vec{p}+\vec{q}) \times (\vec{p}-\vec{q})|}$

$= \pm(-\hat{i} + \hat{j} + \hat{k})$

$|a| + |b| + |c| = 3$

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